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Edgeworth Price Cycles and Focal Prices: Computational Dynamic Markov Equilibria

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Abstract

Motivated by the apparent discovery of Edgeworth Cycles in many retail gasoline markets, this article extends the theory of Edgeworth Cycles along several key dimensions, including models of fluctuating marginal costs, differentiation, capacity constraints and triopoly. A computational approach to search for Markov perfect equilibria is taken. Edgeworth Cycles are found in equilibrium in many situations, and the shape of the cycles are found to carry information about underlying competitive intensity. Cycles in triopoly exhibit interesting coordination problems such as delayed starts and false starts.

1 Introduction

Recently, there has been a great deal of interest from economists about the high-frequency price cycles discovered in a number of retail gasoline markets in Canada, the United States, Australia, New Zealand and several European countries (Noel (2007a, 2007b) Eckert (2002, 2003, 2004), Wang (2005a, 2005b), Allvine and Patterson (1974) and Castanias and Johnson (1993)). The price cycles are surprising at first glance...they are rapid, tall, and sharply asymmetric. Starting from a relatively high level of prices, prices at virtually all stations within a market fall by a small amount each day, typically over the course of a week or a month. Then when markups become sufficiently

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low, firms suddenly and almost simultaneously increase prices back to the original high price, before prices start to gradually fall again. Many of these authors have argued that the price cycles are the theoretically possible (but in practice, seemingly implausible) Edgeworth Cycles of Maskin and Tirole (1988).

To see the visual similarity between the empirical cycles and the theoretical Edgeworth Cycles, compare Figures 1 and 2. Figure 1, taken from Noel (2007b), shows retail prices for a representative major branded station and a representative independent along with the wholesale ("rack") price for the city of Toronto during 2001. The interval between consecutive data points is just 12 hours. The graph clearly shows the asymmetry in retail price changes. When a station increases price, it does so by 13% on average and others follow the increase almost immediately. In contrast, when a station decreases price, it does so by less than 2% per period and the overall decrease is spread out over many periods. The pattern is very similar to that in a theoretical Edgeworth Cycle, which is depicted in Figure 2.[1] I defer to these papers for evidence that the empirical cycles are consistent with Edgeworth Cycles.

—————————— *insert figure 1 about here (immediately above figure 2)* ——————————

—————————— *insert figure 2 about here (immediately below figure 1)* ——————————

A common and valid criticism of the empirical papers, however, is that the market for retail gasoline does not conform perfectly to the setting of the theoretical Maskin and Tirole model. Therefore it is not clear that Edgeworth Cycles can even exist in retail gasoline markets. The standard Maskin and Tirole model that generates Edgeworth Cycles assumes a dynamic, symmetric Bertrand duopoly game with perfectly homogeneous goods.[2] The market for gasoline, however, is

[1] I discuss the mechanism in more detail below. The figure is drawn for a specific demand example with zero marginal costs.

[2] Maskin and Tirole also considered a capacity constrained version of the model.

not perfectly homogeneous. It is not a duopoly. Marginal costs are not constant, firms are not identical, and so on.

The strongest concern perhaps relates to the duopoly assumption. With two firms, it is straightforward in the Edgeworth Cycle model that when one firm raises its price to the top, it is in the best interest of the other to follow by increasing its price just below that of the first firm. This is preferable to pricing at marginal cost or setting any intermediate price. However, when there are three or more firms, and one firm raises its price to the top, is it still true that the other firms necessarily follow? Or would they continue to compete for market share at the lower prices instead? If the latter, it would remove the incentive for the first firm to raise prices to begin with, and might prevent any cycle from occurring at all. And if that is true, the empirical cycles cannot be Edgeworth Cycles. On the other hand, if Edgeworth Cycles can still exist with three or more firms, what properties should we expect the cycles now to have? Do they still match the empirical cycles?

With many authors now claiming to have found empirical examples of Edgeworth Cycles, there is an urgent need to revisit the theory and extend it to include more of these real world complications. Fluctuating marginal costs, markets with more than two firms, and mild amounts of differentiation, for example, are all common in retail gasoline. If theoretical Edgeworth Cycles are not robust to these extensions, it would call the results of these empirical papers into question. If Edgeworth Cycles are robust to some extensions but not others, then it would then be useful to know what matters, to help inform the search for cycles in new markets. We can ask some rudimentary questions: In which markets is it most likely that researchers can observe cycles? When they are observed, can we say anything about the level of competition in cycling markets relative to similar noncycling ones? And finally, do their shapes adapt to environment? In other words, are the cycles homogeneous objects or do their shape carry an additional competitive signature?

To answer these questions, I extend the theory of Edgeworth Cycles to include several key complexities common in real world cycling markets. The first major departure from the Maskin and Tirole model is the inclusion of fluctuating marginal costs. While this is done in large part for technical reasons as explained in the next section, it is also of economic interest in its own right, as fluctuating marginal costs are endemic in gasoline markets. Under the fluctuating marginal cost model, I revisit the standard Bertrand duopoly and capacity constrained versions of it. I also consider a simple model of product differentiation and look at asymmetric equilibria in addition to the standard symmetric ones. When I find cycles, I pay special attention to how each model's parameters affect their shape. The results support that Edgeworth Cycles are a relatively robust construct under fluctuating marginal costs and across a spectrum of situations. Cost shocks in particular do not have a destabilizing effect on the cycles. Cycles can be generated in homogeneous goods markets, in markets with mild but not strong differentiation, and in markets with mild but not strong capacity constraints. Demand levels, elasticities, and discount factors have little to do with cycle existence but rather influence the shape of the cycles and in interesting ways. In empirical practice, both existence and shape can give us an unique view into the nature of competition in studied markets.

The second major departure is the study of a triopoly model, which yields one of the most important sets of findings in the paper. First and foremost, cycles are not restricted to only duopoly settings but are still very feasible under triopoly. But now important coordination challenges arise that did not exist in the two firm model – delayed starts and false starts. Delayed starts occur when competing firms do not immediately follow the price increase of the first firm, stranding the first firm at the top of the cycle for multiple periods. False starts occur when the first firm abandons its attempt to raise prices altogether, after waiting too long for others to follow it to the top of the cycle. Delayed starts and false starts are an important part of the equilibrium cycle process and have real consequences on consumer welfare. Since they make it more costly for a firm to be first to raise its price, the bottom of the cycle becomes lower and firms stay there longer. Average prices fall. While the negative correlation between prices and the number of firms is not surprising, the mechanism by which this works is new and unique. The result gives support and interpretation to researchers who have recently begun to find evidence of false starts in several cycling retail gasoline markets in Canada (Atkinson (2006)) and Australia (Wang (2005c)).

In order to investigate many different scenarios, I employ a computational dynamic programming algorithm to search for equilibria. As with all computational approaches to equilibrium search, there are admittedly disadvantages relative to an analytical approach. For example, with computational equilibria – even the very many I have here – one cannot construct formal sufficiency conditions or write down a complete characterization of all possible equilibria. This is beyond my intended scope. The main goal of this paper is to see if Edgeworth Cycles can plausibly exist under a large number of competitively different situations beyond the basic Bertrand duopoly, and to examine their properties when they do. The computational approach gets a lot of mileage in this regard. I identify a broad range of situations in which Edgeworth Cycles *can* thrive and cast light on some surprising new cycling phenomena, which will be useful right away for informing empirical research in the field. One of the most notable in this regard is the finding of delayed and false starts in the triopoly model.

To date, cycles believed to be Edgeworth Cycles have been found in retail gasoline markets in Canada, the United States, Australia, New Zealand, and several European countries, and also in U.S.-based internet auction markets.[3] Why they have not been detected elsewhere remains an open question, which I address theoretically here. Given my findings, one might imagine that with new technologies creating increasingly real-time markets (in electricity, long distance telephone, internet shopping, etc.) where relatively homogenous products, frequent purchases, and low switching costs are the norm, we may yet see increasing numbers of discoveries of Edgeworth-like cycles. New, higher-frequency data in the hands of researchers will also make the naturally high-frequency cycles easier to detect. This article is a framework to expand our understanding of where and how Edgeworth Cycles can occur and how to interpret them.

The paper is organized as follows. Extensions of the duopoly model, including differentiated goods and capacity constraints, are discussed in Section 2. The triopoly model and discussion of its unique coordination problems appears in Section 3. All models in the paper include the feature that marginal costs fluctuate. Section 4 concludes.

2 Duopoly under Fluctuating Costs

In their original paper, Maskin and Tirole (1988) work with a symmetric Bertrand duopoly, in which homogeneous-goods firms set prices alternately. They restrict their attention to Markov strategies, i.e. strategies that depend only on the payoff relevant state. Examining Markov strategies is useful

[3] See Feng & Zhang (2005) and Zhang (2006) find Edgeworth Cycles in internet auction markets, the first such finding outside of retail gasoline. For Edgeworth Cycles in an experimental setting, see Kruse et al. (1994).

in that it captures the more plausible, simpler strategies, and eliminates a multitude of complicated and often unreasonable equilibria that would still be allowed under unrestricted supergames.[4] I will maintain the Markov assumption throughout the article. The equilibrium concept is that of Markov Perfect Equilibrium (MPE), and is a refinement of Nash equilibrium.

Maskin and Tirole (1988) show in their model that two sets of MPE are possible – focal price equilibria and "Edgeworth Cycle" equilibria. Focal price equilibria are characterized by constant prices over time. Firms tacitly colluding and all charging the monopoly price in each period is an example. Edgeworth Cycle equilibria, in contrast, take the form of an interesting and sharply asymmetric price cycle that is repeated over and over. The mechanism of the cycle is as follows. Starting from a high price, firms repeatedly undercut one another and steal the entire market from its competitor (since goods are homogeneous). Once prices have fallen all the way to marginal cost, undercutting ceases, and firms play a war of attrition with each firm mixing between raising price back to the top of the cycle ("relenting") and remaining at marginal cost. Each would prefer the other to relent first, since the firm to relent second benefits most with higher priced sales in an earlier period. When one firm relents back to the top of the cycle, the other immediately follows but to a price just below that of the first firm, and a new round of undercutting begins.

I now depart from the Edgeworth Cycle duopoly framework of Maskin and Tirole (1988) by adding fluctuating marginal costs. In this section, I reconsider the homogeneous goods case, and then extend the model to incorporate differentiated goods and the presence of capacity constraints.[5]

Assume two infinitely-lived profit-maximizing firms compete in a homogeneous Bertrand pricing game by setting prices in an alternating fashion – one firm sets its price in even periods and the other in odd periods – and once set, the price for that firm is fixed for two periods. Therefore, if firm 1 adjusts its price in period t, $p_t^1 = p_{t+1}^1$ and $p_t^2 = p_{t-1}^2$. Prices are chosen from a discrete price grid. Marginal cost, c_t, is also allowed to vary over time, and is chosen by nature from a discrete

[4]See Maskin & Tirole (2001) for a discussion of the advantages and limitations of working with Markov strategies.

[5]The concept of Edgeworth Cycles originally dates back to Edgeworth (1925) who considers two identically capacity constrained firms. Edgeworth postulated that after undercutting brings firms close to their capacity constraints, one could raise price and profitably serve the residual demand. Notably, Maskin and Tirole show that cycles can exist under both capacity-constrained and unconstrained models.

cost grid with support $[\underline{c}, \overline{c}]$. Each firm earns current period profits of

$$\pi_t^i(p_t^1, p_t^2, c_t) = D^i(p_t^1, p_t^2) * (p_t^i - c_t) \quad (1)$$

where D^i is the standard Bertrand formulation.

The strategies of each firm are allowed to depend only on the payoff-relevant state in each period. Therefore, a firm's strategy depends only on the price set by the other firm in the previous period, current demand, and current marginal cost which it learns prior to setting its price. The Markov Perfect equilibrium strategies are given by R^1, R^2, where $(p_t^1)^* = R^1(p_{t-1}^2, c_t)$, $(p_t^2)^* = R^2(p_{t-1}^1, c_t)$ and $p_{t-1}^j$is the price chosen by firm j is period $t-1$ which remains in effect in period t.[6]

Let $V^1(p_{t-1}^2)$ be the firm 1's value function when firm 2 adjusted its price to p_{t-1}^2 in the previous period, firm 1 adjusts its price in the current period, and the current marginal cost c_t is not yet known. Let $W^1(p_{s-1}^1)$ be firm 1's value function when it has set price p_{s-1}^1 in the previous period, firm 2 is about to adjust its price, and the current cost is not yet known. V^1 and W^1 can be written as

$$V^1(p_{t-1}^2) = \mathop{\mathrm{E}}_{c}\left(\max_{p_t}\left[\pi_t^1(p_t, p_{t-1}^2, c_t) + \delta_1 W^1(p_t)\right]\right) \quad (2)$$

$$W^1(p_{s-1}^1) = \mathop{\mathrm{E}}_{c}\left(\mathop{\mathrm{E}}_{p_s}\left[\pi_s^1(p_{s-1}^1, p_s, c_s) + \delta_1 V^1(p_s)\right]\right) \quad (3)$$

and similar equations are found for V^2 and W^2. The firm-specific discount factor is δ_i. The inside expectation in W^1 is taken with respect to the distribution of R^2 and both the outside expectation in W^1 and the expectation in V^1 is taken with respect to the distribution of c.[7] To choose the best response price, given the current rival price p_{t-1}^2 and current cost c_t, firm 1 maximizes $\pi_t^1(p_t, p_{t-1}^2, c_t) + \delta_1 W^1(p_t)$ (i.e. V^1 without the expectation.) Firm 2 acts in a similar way.[8] Note that the standard Maskin and Tirole (1988) model can be recovered from the current

[6]See Maskin & Tirole (2001) for general properties of MPEs (not simply with alternating moves games).

[7]This formulation implicitly assumes there is no persistence in c, a point to which I will return.

[8]Eckert (2004) uses a dynamic homogeneous goods model and marginal costs with a two-point support to examine the stability of focal price equilibria.

setup by setting $\delta_1 = \delta_2$, and $c_t = c$ for all t.[9] Since marginal costs are constant and known in Maskin and Tirole, the R^i does not depend on c and the expectations in V^i and W^i over c would vanish in their model.

A significant departure of the model presented here is the inclusion of fluctuating marginal costs. This is in part motivated by the degree of wholesale price volatility common to gasoline markets, and my results will check if this has a destabilizing effect on the cycles. But there is also an important technical reason for including fluctuating marginal costs. Because I wish to explore many different competitive environments, I employ a computational dynamic programming algorithm to solve for the value functions V^i and W^i and the best response functions R^i. This would normally present a challenge because focal price and Edgeworth Cycle MPE involve mixed strategies and a computational algorithm based on pure strategies will not converge. However, the use of a simple stochastic marginal cost process simplifies the computation by ensuring a pure strategy pricing equilibrium always exists.[10]

To get a rough sense for how this works, imagine in a constant cost world and in response to some price, the equilibrium involves a firm randomizing between price p_H with probability $\frac{1}{2}$ and p_L with probability $\frac{1}{2}$. In the fluctuating cost world, the firm does not randomize but instead takes a new draw c_t from the marginal cost distribution in the current period. If that draw is above the median, the firm in equilibrium will choose p_H and if below, it will choose p_L. Effectively, the current draw of marginal cost gives the firm a single pure response. I use 2000 grid points in a marginal cost band $[\underline{c}, \overline{c},] = [0, 2]$, to replicate any choice probability to within $\frac{1}{2} * \frac{1}{2000} = 0.00025$ and avoid convergence problems. This does not mean, however, that my results replicate the constant marginal cost case. On the contrary, marginal costs fluctuate within a sizeable band, are payoff relevant, and have their own direct impact on own and competitor profits.[11] As a result it is not uncommon for firms to have two or three possible optimal responses which depend on its cost draw. This is a departure from the constant cost model where mixing generally occurs at one

[9] Eckert (2003) performs an analytic extension of the constant cost model by allowing the firms to split the market unevenly at equal prices. I performed simulations of these models as well and find Edgeworth Cycles with shapes consistent with Eckert. They are not presented for brevity sake.

[10] The equilibrium need not be an Edgeworth Cycle equilibrium.

[11] True mixing probabilities keep the competitor indifferent across the firm's choices. Competitors are not indifferent here across the possible actions taken in response to payoff relevant cost shocks.

place in the cycle and only between two prices.

In exchange for this computational accuracy, some simplification of the cost process is necessary. For example, current period marginal cost is assumed independent of previous draws, the distribution is assumed uniform and finally, marginal cost can change each period although any given price changes every two. These simplifications are taken to avoid significant dimensionality problems in the technical computation, as explained in the footnote.[12]

The system is converged when fixed point vectors V^i and W^i are found. At each iteration k, I update all four value functions as follows. To update V^1 and W^2, I calculate the best response of firm 1, $(p^1)^* = R^1(p^2, c)$, to every possible prior period price p^2 and cost realization c. I then calculate the net present value of profits for each firm (given each p^2, c, $R^1(p^2, c)$, and $V^2_{k-1}(p^1)$ and $W^1_{k-1}(p^1)$ from the previous iteration). Next, for each p^2, I calculate the expectation over c of each firm's profits. This expectation is the new $V^1_k(p^2)$ and $W^2_k(p^2)$. I update V^2 and W^1 similarly.[13]

As mentioned, the computational approach cannot guarantee all possible equilibria will be found. In the Maskin and Tirole model, multiple Edgeworth Cycle equilibria and multiple focal price equilibria exist. Therefore I test a wide range of starting values in each scenario in search of multiple equilibria. Interestingly, however, I routinely converge back to the same equilibrium each time regardless of the starting values attempted, except as noted. In particular, I have not replicated both an Edgeworth Cycle and focal price equilibria in the same scenario. Whenever both Edgeworth Cycles and focal price equilibria might be expected (for example, with very thin cost bands and assumptions close to Maskin and Tirole), all starting values still give the same equilibria – that of Edgeworth Cycles. This suggests that given an initial disequilibrium situation, firms tend to gravitate more easily to Edgeworth Cycles than to focal prices. Since the goal is to show Edgeworth

[12] In the current setup, V^i and W^i depend only on previous price and so are $\#_p$ vectors, where $\#_p$ is the number of points on the price grid. Adding persistence in marginal costs – perhaps as a simply random walk process – means that V^i and W^i must each contain $\#_p * \#_c$ elements (and must converge elementwise), where $\#_c$ is the number of points on the cost grid. In examples that follow, $\#_p$ is 20 but $\#_c * \#_p$ is 40,000. Also, with any distribution other than uniform, either a greater $\#_c$ would be necessary to achieve the same 0.00025 standard error, or the cost grid would have to be spaced unevenly with more points near the peak of the distribution. The choice of the specific cost process, however, will not affect the qualitative results that follow.

[13] Pakes and McGuire (1994, 2001), Ericson and Pakes (1995), Pakes (2000) and others suggest techniques for reducing computational burden such as using a polynomial approximation for the value function and making efficient use of symmetry. Because of the discrete and jumpy nature of the best response functions I describe below (and resulting "waviness" of the value function), I choose to use the precise, but slow, algorithm in the text. I use symmetry between firms to reduce the burden where possible, but since most specifications are asymmetric in nature, this is of limited value.

Cycles can thrive under a variety of environments, the tendency for the computational method to gravitate towards Edgeworth Cycles when there are multiple equilibria is not problematic. In fact, the simulations are very successful in finding Edgeworth Cycles. For consistency, all reported results use starting values for V^i and W^i that would be the outcome in a single period static game (i.e. if discount factors were zero) except as noted.

2.1 Symmetric Duopoly Model

I begin with the symmetric homogeneous Bertrand duopoly model but with fluctuating marginal costs. The simulations show cycles are easily generated when there are fluctuating costs under all reasonable parameter values.

I fix a particular example for discussion now, and examine variation in models and parameter values shortly. Assume a linear demand curve given by $D(p) = a - bp$, where $a = 20$ and $b = 1$. Prices are chosen from a discrete price grid $p^i = \{x\},\ x = 0..20$. Marginal cost in each new period is randomly drawn from a discrete uniform cost grid in the range $c_t = \{x/1000\},\ x = 0..2000$. Also, set the discount factor $\delta_i = 0.95$ for each firm. In the top panel of Figure 3, I report the equilibrium best response functions $R^i(p^j_{t-1}, c_t)$ for this case, and in the bottom panel I report the equilibrium price paths over 40 periods.[14]

——————— *insert figure 3 about here* ———————

Before interpreting results, it is a useful digression to explain how to quickly read the best response figures. Consider firm 2's best response function, depicted with dark circles in the figure. A circle immediately below the 45° line represents an undercut of one notch on the price grid by firm 2. A circle placed further below the 45° line represents a more aggressive undercut. Matching firm 1's price is represented by a circle directly on the 45° line and when firm 2 responds by raising

[14] These are best response functions and not best response correspondences because there is a unique best response to each price *conditional* on the specific value of marginal cost. (Except for the zero probability event that the cost draw is exactly such that a firm is indifferent between two price responses.) Because the price grid is discrete, the displayed best response functions are also discrete. I have, however, connected the dots in the price path figures to facilitate presentation.

its price back to the top of the cycle, which I call "relenting", we observe circles far above the line. Had firm 2 wanted to respond by raising its price only slightly instead, which I call "stepping up" (and not observed in this example), we would see a circle only slightly above the line. When firm 2 will respond with one of two or more actions (depending on realized marginal cost), multiple circles appear on the same vertical line.[15] Reading firm 1's best response function is similar: undercuts are represented by hollow squares to the left of the 45° line, relenting and stepping up are to the right, and multiple squares on the same horizontal line represent multiple possible best responses, depending on cost.

The figures clearly show cycles in equilibrium with changes in firms' behavior along the path of the cycle. First, notice that prices at the top of the cycle begin substantially *above* the static monopoly price.[16] Clearly a dominated strategy if this were a one-shot static game, during an Edgeworth Cycle this is preferable and allows firms to spend more time near the peak of the profit distribution, which is maximized at the monopoly price. The undercutting then proceeds in an orderly fashion, one notch at a time, through the most profitable prices.

However, once far enough off the peak of the profit function, orderly price undercutting yields to aggressive price cutting. One of the firms undercuts by three or four notches at once, quickly pushing the cycle deep into the less profitable region. The benefit is that it puts pressure on its competitor to accept the costly task of relenting in the following period.[17] If the other firm does not quickly relent, the first firm will either relent itself or switch to a *passive* strategy instead. In the latter case, it matches price instead of undercutting, accepting a split of the market at unprofitable prices, and passing the turn back to its opponent. Eventually, a high enough cost draw will cause one of the firms to relent. The other follows immediately and undercutting recommences.

Relative to a repeated one-shot Bertrand game, the average market price consumers face is relatively high. At 8.5, it is close to the average static monopoly price of 10.5 and well away from

[15]For readability, when there are two or more possible price responses depending on the realized value of the cost draw, I do not report the exact probabilities (or scale the size of the symbols) in the graphs. It is always the case that undercutting occurs at lower marginal costs than matching, and matching occurs at lower marginal costs than relenting or stepping up.

[16]Define the static monopoly price as the average of the monopoly prices that would occur in one-shot games. In this case, it is 10.5 (i.e. 10 with probability 0.5 and 11 with probability 0.5.)

[17]Although excluded from the diagram to reduce clutter, the probability of a firm relenting increases as the price falls from 4 to 1, and is certain from a price of 0. (Since 0 is not a response to any price, though, it does not occur on the equilibrium path.)

the highest static competitive price of 2.

2.2 Elasticities and Discount Factors

I performed additional trials for robustness on linear demand curves $D(p) = a - bp$ and allow parameters to change. Cycles are easily generated for all tested $\bar{c}b < a < \infty$ and $-\infty < b < 0$.[18] This is not surprising because the gain to undercutting comes predominantly from stealing the existing consumer base and not by creating new consumers. Consistent with Maskin and Tirole, researchers cannot exclude Edgeworth Cycles on the basis of aggregate demand alone in the fluctuating marginal cost case.

——————— *insert figure 4 about here* ———————

Aggregate elasticity does impact the shape of the cycle however. The best response functions with relatively elastic demand curves show that firms are *less* aggressive in undercutting, more often proceeding by one notch at a time into lower prices. This is because firms can serve a relatively larger market at low prices. The best response functions and cycles for the relatively more elastic case $a = 15$, $b = 0.5$ are shown in Figure 4 for comparison. With less elastic curves instead, simulations show more aggressive undercutting and cycles that are relatively more rapid and less asymmetric.[19]

For any given set of demand parameters, the simulations easily generate Edgeworth Cycles under fluctuating costs for all tested pairs of discount factors $\delta_i \in (0, 1)$, whether equal or unequal.[20] With a wide enough cost band (to contain several points on the price grid), there is always a cost draw low enough to make a undercut near $\bar{c}$ profitable, and a later cost draw high enough to ensure a

[18]For linear demand curves, $a/b > \bar{c}$ is required for demand to support a price above marginal cost. Eighty combinations were tested with $0.03 < a < 60$, $0.01 < b < 10$.

[19]For comparisons, I used demand curves pivoted around a particular point (for these examples, (10,10)). I also simulated parallel shifts in demand and pivots around the quantity intercept with descriptively similar results. Pivoting around the price intercept yields a set of identical best response functions since the elasticity at a given price is unchanged.

[20]I experimented with $\delta_i = \{0.01, 0.05x, 0.99\}, .x = 1..19$. Even when $\delta_i = 0$, Edgeworth Cycles is still one possible type of equilibrium, since when prices are at marginal cost, a firm is indifferent across all $p \geq c$. Any $p > c$ will create a cycle.

relent.

Discount factors play an important role in shaping the cycle. The best response functions show that with lower discount factors (more weight on current profit), the first undercut from the top of the cycle is relatively closer to the static monopoly price. Undercutting proceeds more slowly and matching is much more likely at low prices. The shape reflects this. The downward slope of the cycle loses its concavity and becomes linear until, near the bottom, price matching adds some convexity. Relative to higher discount factors, the cycles are shifted vertically downward, are longer in duration, and are more asymmetric.[21]

For a concrete example, consider the case of $\delta_i = 0.5$ (not shown) with demand parameters as above. The first undercut is to 11 or 12, slightly above but relatively closer to the static monopoly price. Undercutting proceeds by one notch at a time all the way down to a price of 2, when matching during the war of attrition begins. Consumers are also better off: average price is 5.8 when $\delta_i = 0.5$, 44% below the static monopoly price and 31% below the $\delta_i = 0.95$ case.[22]

2.3 Asymmetric Strategies

Many theoretical expositions focus on symmetric equilibrium strategies. Clearly when firms are meaningfully different – different levels of consumer loyalty, different locations, different capacity constraints – equilibria will exist where firms do not follow identical strategies. But even if firms are identical, asymmetric Edgeworth Cycle equilibria still exists, as I show here. Theory does not tell us which equilibrium will be reached, but it is worth noting that symmetric equilibria tend to be less stable. Small random perturbations in the symmetric equilibria that affect firms differentially can create strategy differences that cause a move towards a stable but asymmetric equilibria.

——————— *insert figure 5 about here* ———————

To find asymmetric equilibria computationally, I make a small random perturbation in the

[21] It is interesting to note that while economists describe such firms (with low δ) as less patient, some non-economists may view these firms as being *more* patient since they routinely wait out a war of attrition longer than high-δ firms.

[22] When $\delta_i = 0.99$, average price rises to 9.2, 11% higher than the $\delta_i = 0.95$ case.

starting values in V^i for one of the firms. The results of this experiment are shown in Figure 5 for the example introduced above. Edgeworth Cycles still exist, but this simple change creates a cycle that operates quite differently. The same firm – in this example, firm 1 – now relents first each time. (Note the absence of a horizontal line of circles in the upper left.)[23] It does this because in this equilibrium it knows firm 2 will never relent first, and firm 2 never does because it knows firm 1 always will. As a result firm 2 plays an aggressive strategy at high and moderate prices – starting with its first undercut to a price near the static monopoly price, rather than far above it. Since firm 1 will surely be first to relent, firm 2 has less incentive to delay the next cycle trough into the future. After a short range of one-notch undercutting, firm 2 aggressively undercuts to a low price to induce a quick relent. If firm 1 does not immediately relent, firm 2 switches to a passive strategy, either matching price or instead simply "stepping up" its price just above that of firm 1 (as shown by circles just above the 45° line). Though not observed on the equilibrium path, the step up strategy is important for firm 2 to avoid relenting itself. We will see this overall pattern of behavior emerge whenever we look at cases involving meaningfully different firms.

The behavior gives a different shape to cycle. Compared to its symmetric counterpart in Figure 3, the resulting cycle in the asymmetric case is more rapid, smaller in amplitude, and less asymmetric. Similar Edgeworth Cycle equilibria are easily generated for all tested parameter values $\bar{c}b < a < \infty$, $-\infty < b < 0$, and $0 < \delta_i = \delta_j < 1$. Comparisons across symmetric and asymmetric equilibria hold in each case.

Given prices are strategic complements, there is a profit advantage to the follower. Firm 2 enjoys profits 50% higher than firm 1, and 24% higher than in the symmetric case. Firm 1's profits fall by 17%. Consumers are also better off because of the aggressive play of firm 2, with an average price of 7.7, or 10% lower than in the symmetric case.

2.4 Capacity Constraints

Many industries, including gasoline markets, have capacity constraints on at least some firms. This can diminish the firm's incentive to undercut if it can no longer serve the entire market at the new lower price. Moreover, residual demand is left over when an opponent's constraint binds so a firm

[23]There is another mirror-image equilibria with firm 1 and firm 2 reversed.

may prefer a relatively high price for this reason. In this section I add capacity constraints to the fluctuating costs model. Demand is recast as

$$D^i(p_t^1, p_t^2) = \begin{cases} \min\{K^i, D(p_t^i)\} & \text{if } p_t^i < p_t^j \\ \min\{K^i, D(p_t^i) - \min[K^j, \frac{1}{2}D(p_t^j)]\} & \text{if } p_t^i = p_t^j \\ \min\{K^i, \max[0, D(p_t^i) - \min[K^j, D(p_t^j)]]\} & p_t^i > p_t^j \end{cases} \tag{4}$$

for $i \neq j$, where K^i is the maximum output, or capacity, of firm i. Capacities are exogenously given.[24] Marginal cost is assumed to be c_t – which varies – for all units up to K^i and is equal to infinity thereafter. To fix an example for discussion, I assume $\delta^i = 0.95$, $a = 20$ and $b = 1$ as earlier, and perform trials using all meaningful integer values of K^i.

Symmetric Capacity Constraints

First consider the case of symmetric capacities, $K^1 = K^2$. Intuition suggests that when capacity constraints are very loose, Edgeworth Cycles should exist as they did in the unconstrained version and at some point, when they are very tight, they should not. Consistent with this intuition, the simulations easily generate Edgeworth Cycles for this example for all $K^i \geq 10$. But for $K^i < 10$, only focal price equilibria appear. I find the same patterns described below for all $\overline{c}b < a < \infty$, $-\infty < b < 0$, and $0 < \delta_i = \delta_j < 1$ tested, with obvious changes in the range and threshold value of K.

——————— *insert figure 6 about here* ———————

——————— *insert figure 7 about here* ———————

[24]Some models of endogeneous capacity choice in other contexts include Kreps and Scheinkman (1983), Kovenock and Roy (1998), and Reynolds and Wilson (2000).

What is interesting is how changes in capacities affect the shape of the cycle. Simulations show that K^i falls toward 10, the undercutting phase becomes more linear and longer. Edgeworth Cycles become less rapid and more asymmetric. The reason is that when a firm's capacity-constrained opponent undercuts the firm in the following period, it leaves residual demand for the firm. Having made a smaller undercut two periods earlier allows the firm to serve that residual demand at a higher price. As a result, multiple notch undercuts are increasingly rare and the undercutting phase slows. In Figure 6, I show the example of $K^i = 10$. The sales-weighted average price in this case is 8.27, only 3% below its unconstrained counterpart. Smoother undercutting tends to lower the average, but the fact that firms no longer set very low prices on the equilibrium path and the fact that the higher priced firm still makes sales tends to raise it.

Below $K^i = 10$, the best response functions reveal focal price equilibria only. The example of $K^i = 9$ is shown in Figure 7. In the example, the focal price is $p^f = 7$ and firms carry 28% excess capacity.[25] Excess capacity is necessary to credibly threaten to punish defections from the focal price. This appears as a square box in the best response functions graph. Firms threaten to retaliate by undercutting the undercutter and then engaging in a war of attrition until some firm restores the focal price.

As K^i decreases below 9, the simulations show two opposing effects on the focal price. First, it becomes more difficult to punish defections from a given focal price since firms have less capacity. This works toward a lower focal price and firms producing closer to capacity. Second, when capacities are tighter, the market-clearing price that would occur with full capacity production rises. The former effect dominates with high K^i and the latter dominates at lower K^i. In the example shown, the focal price reaches its minimum value of 6 at $K^i = 8$. Excess capacity is 13%. By the time $K^i \leq 5$, excess capacity is zero and the now full capacity market clearing price continues to rise.

Asymmetric Capacity Constraints

Now consider the case when only one firm (firm 2) is constrained. This is motivated by the fact

[25] This is consistent with Maskin & Tirole (1988) who find excess capacity is built to support focal prices in the constant costs case. The result is also similar to Benoit & Krishna (1987) and Davidson & Denekere (1990) who show that excess capacities can support higher collusive prices in pricing supergames. See Brock & Scheinkman (1985), Rotemberg & Saloner (1986), Haltiwanger & Harrington (1991) and Bagwell & Staiger (1996) for discussions of maximum sustainable cartel prices in various contexts.

that major gasoline retailers have more overall capacity than do independents. I find that for all integer values of $K^2 > 0$, I can easily generate Edgeworth Cycles. Results hold for all $\bar{c}b < a < \infty$, $-\infty < b < 0$, and $0 < \delta_i = \delta_j < 1$ tested with obvious changes in the range of K. There are interesting changes in the shape of the cycle as capacity constraints tighten, however. In fact, if tight enough, one might overlook the cycle entirely.

——————— *insert figure 8 about here* ———————

The simulations show that with firm 2 is constrained, it is firm 1 that relents first on each cycle.[26] It is not as costly for firm 1 to relent because it has more capacity to serve the residual demand at high prices.

For relatively weak constraints, the best response function is similar to that in Figure 5. But as constraints on firm 2 tighten, the cycle becomes more rapid, smaller in amplitude, and less asymmetric. Firm 2 is increasingly more aggressive in undercutting as firm 1 is increasingly likely to relent from moderate prices. I report the case of $K^2 = 7$ in Figure 8, showing the rapid, low amplitude cycle. By the time $K^2 = 2$, even a careful observer could easily overlook them. Firm 1 always sets price at 9 or 10 depending on cost, and firm 2 always sets a price of 7. But these are not different "focal" prices – it is a "Hyper-Edgeworth Cycle". Firm 2 could have sold to capacity by matching, but then firm 1 would have responded with an undercut and left zero sales for firm 2. Hence, firm 2 undercuts by just enough to force firm 1 to relent on every turn. The hypercycle can be difficult to detect. One indication might be that the two firms charge substantially different prices in equilibrium for what are actually homogeneous goods.

Capacity constraints have expected effects on average sales-weighted prices – prices edge up with lower K^2. At $K^2 = 7$, it is 7.3 but then rises to 9.8 when $K^2 = 3$ and ultimately 10.5 (the static monopoly price) when $K^2 = 0$.

[26]This agrees with the Deneckere & Kovenock (1992) result in their constant-cost, two-period model of price competition, where the large firm endogeneously becomes the price leader.

2.5 Differentiated Products

Most industries are characterized by at least some degree of product differentiation. With differentiation, the incentive to undercut is diminished since some consumers will remain loyal to the opponent's product. The simulations in this section confirm this intuition. I can easily generate Edgeworth Cycles when differentiation is sufficiently weak but not above a certain threshold. In the simple Hotelling model I use to illustrate below, Edgeworth Cycles are only found when firm's loyal consumers represent less than 20% of their consumer base.

Assume consumer tastes are uniformly distributed over a univariate product space that has support [0,1]. Firm 1's product is located at point 0 and firm 2's product is at point 1. Each consumer h has unit demand and receives utility in time t of $v^i - p^i_t - \tau z^{ih}_t$ if she purchases from firm i, where v^i is the intrinsic value of firm i's product, p^i the price charged by firm i, and z^{ih} is her distance in product space to her most preferred product. Let τ is the disutility per unit of distance between the preferred and the purchased product.

If the prices are low enough that all consumers make a purchase, which occurs when $(v^1+v^2) - (p^1_t + p^2_t) \geq \tau$, the market share of firm i is given by

$$s^i_t(p^1_t, p^2_t) = \begin{cases} \frac{1}{2} + \frac{(v^i - v^j)}{2\tau} - \frac{(p^i_t - p^j_t)}{2\tau} & \text{if } \left|(v^i - p^i_t) - (v^j - p^j_t)\right| \leq \tau \\ 1 & \text{if } (v^i - p^i_t) - (v^j - p^j_t) > \tau \qquad i \neq j \\ 0 & \text{if } (v^j - p^j_t) - (v^i - p^i_t) > \tau \end{cases} \tag{5}$$

If prices are high enough that not all consumers are served at those prices, then firm i's share is

$$s^i_t(p^i_t) = \frac{v^i - p^i_t}{\tau} \tag{6}$$

Letting H be the total number of consumers in each period, current period profits to firm i is

$$\pi^i_t(p^1_t, p^2_t, c_t) = H * s^i_t(p^1_t, p^2_t) * (p^i_t - c_t) \tag{7}$$

which is substituted into the equations for V^i and W^i given above. By construction in this model, aggregate elasticity is zero while all consumers are served, and $-\frac{2}{t}$ when not all served.

To fix an example for discussion, assume $v^1 = v^2 = 10$, a price grid of $p^i = \{x\},\ x = 0..v^i = 1..10$, the marginal cost grid as before, and $\delta_i = 0.95$. Consider $\tau \in [1, \infty)$.[27] Note that when $\tau < 1$ a one-notch undercut steals the entire market so that this case is equivalent to a homogeneous goods Hotelling model. For comparison sake, firms in the homogeneous goods version of the Hotelling model (not shown) undercut by one notch at a time through most prices, by two in several, and match at very low prices.

——————— *insert figure 9 about here* ———————

——————— *insert figure 10 about here* ———————

When $\tau > 1$, the market is meaningfully differentiated in that not all consumers will switch in response to a minimum undercut. The case of $\tau = 1.1$ is illustrated in Figure 9. An undercut of one notch steals an additional 45%, of the market in the case, leaving the other firm with 5%.[28] The figure shows a clear Edgeworth Cycle equilibrium. Average sales weighted price is 5.2, well below the static monopoly price of $p^m(v, \tau) = v - \frac{\tau}{2} = 9.45$, but above the average static competitive price of 2.1. Simulations easily generate Edgeworth Cycles for all values of $\tau \in [1, 1.25)$.

With enough differentiation ($\tau \geq 1.25$), however, the simulations can no longer generate Edgeworth Cycles but only focal prices in equilibrium. The case of $\tau = 1.25$, when a one-notch undercut steals an additional 40% of the market, leaving the firm with 20% of its original base, is shown in Figure 10. The focal price is 7 and firms stand ready to punish the defection with a further undercut.

As τ grows above 1.25, there are two opposing effects on the equilibrium focal price. First, it becomes more costly to punish defections from a given focal price since a greater undercut would be needed to impact market share strongly. This works to reduce the sustainable focal price. However,

[27] Trials use $\tau = \{1 + 0.05x,\ 2 + x\}$, $x = 0..10$.

[28] This best response function diagram is identical to the previous $\tau \leq 1$ case. Small differences in the choice probabilities still exist.

consumers are willing to pay a higher price since products are more differentiated. The former effect dominates with lower τ and the latter dominates at higher τ. For example, firms can no longer credibly threaten to respond to an undercut with another undercut (only to match) when $\tau \geq 2$ and the focal price falls to 5. It then gradually rises to 7 by the time $\tau = 6$.[29] Above $\tau = 6$, firms compete against the no purchase option rather than each other and prices gradually fall again. With $\tau \geq 9$, firms do not serve the middle consumers and set monopoly prices of 5 or 6 (depending on cost) thereafter.[30]

Additional simulations find the same pattern of results for all symmetric v^i and δ_i.

Differences in real or perceived differentiation provide a strong possible explanation for why cycles exist in some retail gasoline markets but are absent in others. Barron et al. (forthcoming) recently estimated demand elasticities in the Los Angeles, San Diego, and San Francisco retail gasoline markets, where cycles currently do not exist, and report a median firm-level own price elasticity of -2.1. In contrast, when Wang (2005b) examined elasticities for the city of Perth, Australia, where strong cycles do exist, he found a median firm-level own price elasticity of -7.8. While the gain to undercutting in the California markets would be relatively small, even small undercuts in the Perth market would be very effective in stealing market share.

3 Bertrand Triopoly under Fluctuating Costs

Earlier studies have focused on the Bertrand duopoly model for its analytical tractability. But a highly relevant question for researchers studying empirical cycles in the field is: Can Edgeworth Cycles exist when there are more than two firms? If they cannot, the empirical cycles found in many retail gasoline markets and in internet auctions cannot be Edgeworth Cycles. If they can exist, it is important to understand what they look like with more than two firms and how they differ from the Maskin and Tirole standard. Are they consistent with what researchers find? In this section, I break from earlier work by examining a Bertrand triopoly, while maintaining the assumption of

[29] At $\tau = 2$, a one-notch undercut steals an additional 25% of the market. At $\tau = 6$, it steals only 8%.

[30] Recall that when all consumers are served, the monopoly price is $p^m = v - \frac{t}{2}$ and falls with τ. When not all consumers are served, which requires $\tau > v - c$, the static monopoly price is $p^m = \frac{(v+c)}{2}$ and is independent of τ.

fluctuating marginal costs.[31]

It is clear that when one firm relents in a duopoly setting, the best response for the other firm is to immediately follow, starting the next cycle. But this is not at all obvious when there are more than two firms, since all but one are still competing with each other near the cycle bottom. If they do not follow, the first firm has no incentive to relent in the first place and it is possible that cycles may never occur. This is a common criticism of the empirical literature on Edgeworth Cycles.

The first major result of this section is that Edgeworth Cycles can exist quite readily in a Bertrand triopoly. This finding is robust to a wide range of parameter values and gives support to empirical researchers who claim to have found Edgeworth Cycles in the field. However, the second major result is that important coordination challenges now arise that did not exist in the two firm model. There are delayed starts and false starts. Delayed starts occur when competing firms do not immediately follow the price increase of the first firm, stranding the first firm at the top of the cycle for multiple periods. False starts occur when the first firm abandons its attempt to raise prices altogether, after waiting too long for others to follow it to the top of the cycle. Delayed starts and false starts are an important part of the equilibrium cycle process and have real consequences on consumer welfare, as discussed below.

In the three firm game, each firm can adjust its price every third period and its price is fixed for the following two. Firm 1 adjusts its price in period t, firm 2 in $t+1$, and firm 3 in $t+2$ before returning to firm 1 again. The value functions for firm 1 are:

$$V^1(p^2_{t-2}, p^3_{t-1}) = \underset{c}{\mathrm{E}}\left(\max_{p_t}\left[\pi^1_t(p_t, p^2_{t-2}, p^3_{t-1}, c_t) + \delta_1 W^1(p^3_{t-1}, p_t)\right]\right) \quad (8)$$

$$W^1(p^3_{s-2}, p^1_{s-1}) = \underset{c}{\mathrm{E}}\left(\underset{p_s}{\mathrm{E}}\left[\pi^1_s(p^1_{s-1}, p_s, p^3_{s-2}, c_s) + \delta_1 U^1(p^1_{s-1}, p_s)\right]\right) \quad (9)$$

$$U^1(p^1_{r-2}, p^2_{r-1}) = \underset{c}{\mathrm{E}}\left(\underset{p_r}{\mathrm{E}}\left[\pi^1_r(p^1_{r-2}, p^2_{r-1}, p_r, c_r) + \delta_1 V^1(p^2_{r-1}, p_r)\right]\right) \quad (10)$$

The value function $V^1(p^2_{t-2}, p^3_{t-1})$ is the expected future profits of firm 1 at a time t when it is firm 1's turn to adjust its price, given that firm 2 set price p^2_{t-2} two periods before ($p^2_{t-2} = p^2_{t-1} = p^2_t$), firm 3 set price p^3_{t-1} in the previous period ($p^3_{t-1} = p^3_t = p^3_{t+1}$), and c_t is not yet known. Similarly,

[31] Computational demands preclude models involving more than three firms due to the higher dimensionality of the best response function. However, it is reasonable that findings of the triopoly model will carry forward to models involving a moderate number of firms greater than three.

the value function $W^1(p^3_{s-2}, p^1_{s-1})$ is firm 1's expected future profits at a time s when it is firm 2's turn to adjust its price and $U^1(p^1_{r-2}, p^2_{r-1})$ is its expected future profits at time r when it is firm 3's turn to adjust price. V^2, W^2, U^2, V^3, W^3, and U^3 are similarly defined.

The per period profit function is the standard Bertrand formulation. The lowest priced firm serves the entire market, or if two or more firms have the lowest price, they split the market in halves or thirds accordingly. Again, to fix an example for discussion, let $D(p) = a - bp$, $a = 20$,and $b = 1$. Because of the additional computational demands of the three firms model, I allow for 200 points on the cost grid, $c_t = \{x/100\},\ x = 1..200$.[32]

If we believe that the interval of time between firm i's moves should not change regardless of how many firms there are, then firms should care about its profits three periods hence in the three firm model as it would two periods hence in the two firm model. To adjust for this, I use a discount factor of $\delta_1 = \delta_2 = \delta_3 = 0.967$ in the base case.[33] If instead we believe that the time interval between consecutive price changes by different firms should not change, $\delta_i = 0.95$ would again be used. Results are very similar between the two.

3.1 Cycles in Triopoly

——————— *insert figure 11 about here* ———————

The simulations clearly generate Edgeworth Cycles as an equilibrium in triopoly. An example of the firms' price paths are given in the top panel of Figure 11. The best response functions are multivariate and not shown. Simulations converge to Edgeworth Cycles across all tested parameters $\bar{c}b < a < \infty$ and $-\infty < b < 0$.

In the triopoly example presented, the last firm to raise its price undercuts the previous two to a price of 14 and captures the market. The undercut is to a price substantially greater than the static monopoly price (of 10 to 11). From there, the active firm undercuts the lowest priced firm by one notch on the grid through high and medium prices. Once the minimum price of the other two reaches 6, a firm may undercut by one notch as usual if cost is high, or aggressively undercut

[32] Choice probabilities can be replicated to within 0.0025.
[33] $0.967^3 \cong 0.95^2$

by several notches if the cost draw is low. The aggressive play pressures opponents into relenting earlier. At low prices, the active firm – if it does not relent – responds either by undercutting one notch or matching the lowest price. Eventually, one firm raises its price in an *attempt* to lead all prices back to the top of the cycle.

3.2 Delayed and False Starts

I say "attempt" because immediate following by the other two firms is no longer guaranteed as it was in the duopoly case. Instead, there can be "delayed starts" in cycle resetting, and in some instances "false starts."

A delayed start occurs when a firm must wait more than one turn (three periods) for others to follow it to the top of the cycle. The top panel of Figure 11 shows an example of a delayed start around the third peak. It is easily identified by an extended flat line at the top of the cycle. After a high cost draw and facing low opponent prices, firm 2 is the first to relent to the top. But with a low cost draw in the next period, firm 3 finds it more profitable to undercut further rather than follow firm 2. The result is that firm 2 sits at the top of the cycle and makes no sales for two consecutive turns (six periods in all) instead of the usual one. Longer delays can also occur.

A false start occurs when a firm abandons its effort to reset prices higher altogether and returns immediately to the bottom with the other firms. Two examples of false starts are shown in the bottom panel of Figure 11. They take on the appearance of double peaks along the price path – the first and third main peaks show false starts. (The reader will note the second peak is another example of a delayed start.)

Consider the first false start in the figure. In this case, firm 2 relents first after facing low opponent prices and suffering a very high cost draw. Unfortunately for firm 2, firm 3 and then firm 1 receive favorably low cost draws in the following two periods and – rather than follow – continue to undercut each other. Had its next cost draw been high, firm 2 would have remained at the top for another turn, and we would observe a delayed start. In this example, however, its receives a cost draw low enough that it is more profitable to abandon its position at the top of the cycle and match firm 1's price at the bottom. This action delays the resetting of the cycle but greatly increases the probability (up from zero) that the others will relent first.

In simulations, false starts occurred in 6% of all attempted relents (i.e. all peaks) and delayed starts occurred in an additional 13%.

Average prices fall with more firms, as one might expect, but the mechanism by which this works is interesting. Note that peak prices and trough prices have not changed from the duopoly case. And firms still capture the entire market with a one-notch undercut. The key difference is coordination problems. Delayed starts and false starts make it more challenging and costly to be first to reset the cycle. As a result, firms hesitate in relenting and market prices stay near the band of marginal costs longer. This is easily seen in a comparison of Figures 3 and 11. The average market price on the equilibrium path is now 7.2, 16% lower than in the two firm case.

When $\delta^i = 0.63$ instead, coordination problems worsen.[34] Firms undercut by one notch through even lower prices and extended matching at prices of 1 and 2 are commonplace. False starts and delayed starts are also more common, and the delays are longer. Average market price is lower still at 5.51.

I can also generate cycles in triopoly under mild amounts of differentiation and with relatively weak capacity constraints, as was true in the duopoly case. The qualitative results from the duopoly case carry over.[35]

In summary, I find Edgeworth Cycles are robust to the inclusion of more than two firms. However, in triopoly, the cycle is a bit more fragile. Coordination problems, in the form of delayed starts and false starts, can occur. These make relenting more difficult for firms but benefit consumers through lower average prices.

Notably, the findings of this section lend further support to researchers who claim to have observed Edgeworth Cycles in retail gasoline markets. First, it shows that retail gasoline markets need not be duopolies in order to sustain Edgeworth Cycles. Second, it points out that tell-tale patterns – in the form of delayed starts and false starts – can appear along the path of the cycle. These can be difficult to explain outside the Edgeworth Cycle model.[36] And recent empirical evidence in Canada (Atkinson (2006)) and Australia (Wang (2005b)) now show that these patterns

[34]Comparable to the $\delta^i = 0.5$ two firm case, as $(0.63)^3 = (0.5)^2$.

[35]Although computationally infeasible currently, it would be interesting to model cycles in a market with many weakly spatially differentiated retail outlets under a variety of ownership patterns, while allowing for capacity constraints and both varying costs and demands. These features all occur simultaneously in retail gasoline markets.

[36]Noel (2007a, 2007b) lists and rules out a number of alternate hypotheses of the cycles.

are in fact occurring in numerous retail gasoline markets. This suggests that the empirical prices cycles are indeed being driven by an Edgeworth Cycle process.

4 Conclusion

This article was motivated by the discovery of apparent Edgeworth Cycles in retail gasoline markets in the U.S., Canada, New Zealand, Australia, and in Europe, and a desire to examine several key gaps between the theory and real world experience. In this article, I employ a computational approach to search for Edgeworth Cycles under a wide assortment of competitive models. In a framework that allows for fluctuating marginal costs, I show that Edgeworth Cycles can exist in many scenarios beyond the homogeneous Bertrand duopoly. They can exist in differentiated goods markets when the differentiation is not too strong and in capacity constrained markets provided symmetric constraints are not too tight. They exist under a wide range of market elasticities and discount factors. A key finding is that Edgeworth Cycles can thrive in triopoly markets. In triopoly, however, firms face new coordination challenges – delayed starts and false starts – that do not occur in the two firm model. These have recently been reported as real empirical phenomena. I also find that the shape of the cycles is impacted by the aggressiveness of the firms and this varies across competitive situations in informative ways.

The article takes an important step to understanding the range of environments conducive to Edgeworth Cycle activity. Edgeworth Cycles are robust to many scenarios, but the ability for firms to steal large numbers of consumers with small price changes remains central.

So why do we not see Edgeworth Cycles in even more types of markets – both inside and outside of retail gasoline? It is interesting that in most cases where cycles are newly found, it is because finer and newly available data reveals previously hidden cycles, rather than because cycles are newly formed. In markets where we are sure Edgeworth Cycles do not currently occur, it is an open question which factor most inhibits them. As discussed in Section 2, differentiation is likely an important contributing factor. This can take the form of traditional product differentiation (real or perceived), or work through search costs and benefits. With higher search costs, consumers are less likely to search and even less likely for products purchased infrequently or that account for a relatively small percentage of expenditures. Gasoline retailing might give the closest example to the opposite – relatively homogeneous products, low search costs, a high percentage of expenditures and frequent purchases. Market structure may also be an important factor in inhibiting cycles in many industries, as the triopoly model of Section 3 shows coordination in raising prices becomes an important problem. With many firms in an industry, it is not unreasonable that the free rider problem may become severe enough that, in practice, firms choose not to attempt price increases at all. For example, in many gasoline markets in the U.S., price control is relatively more likely in the hands of individual station dealers than integrated refiner-retailers, relative to other countries, and cycles would be more difficult to generate. Finally, I note that retail gasoline markets are spot markets, but in other industries where pricing schemes like contract pricing or multi-part tariff pricing – which can reduce switching costs – are standard, cycles may be more difficult to sustain. Looking toward the future, however, it is not unreasonable to think we may observe Edgeworth Cycles in even more types of markets outside retail gasoline. New technologies continue to create increasingly homogeneous retail markets (in electricity, long distance telephone, internet shopping, etc.) where relatively homogenous products, frequent purchases, and low switching costs are the norm. Assuming we have data sufficiently fine to reveal high frequency price changes, these kinds of markets would be natural places to find the next new examples of real world Edgeworth Cycles.

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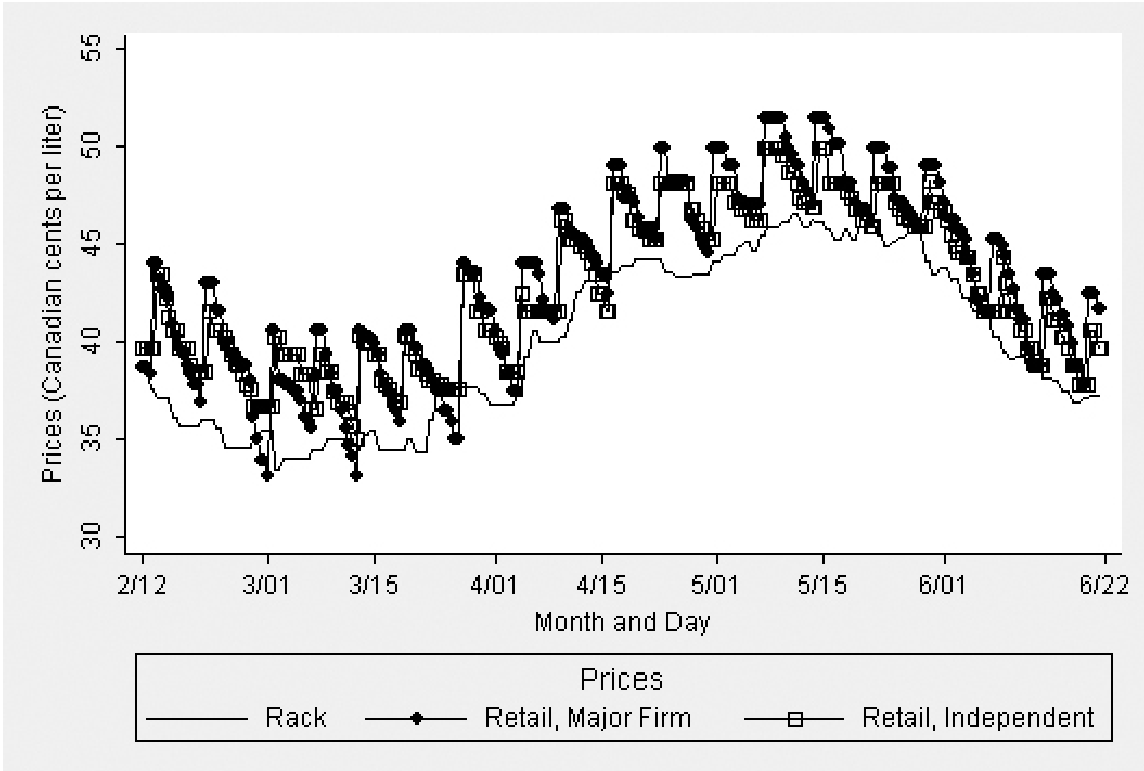

Figure 1. Retail Prices (Major Firm, Independent Firm) and Rack Price

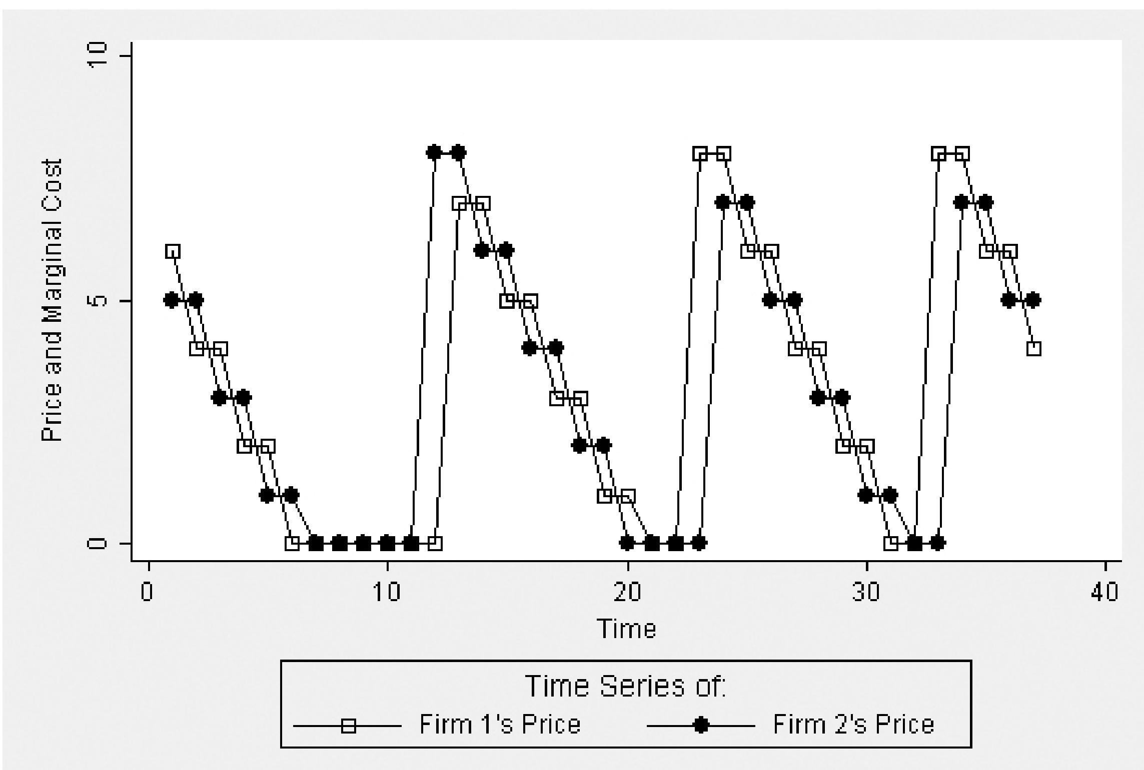

Figure 2.Theoretical Edgeworth Cycle

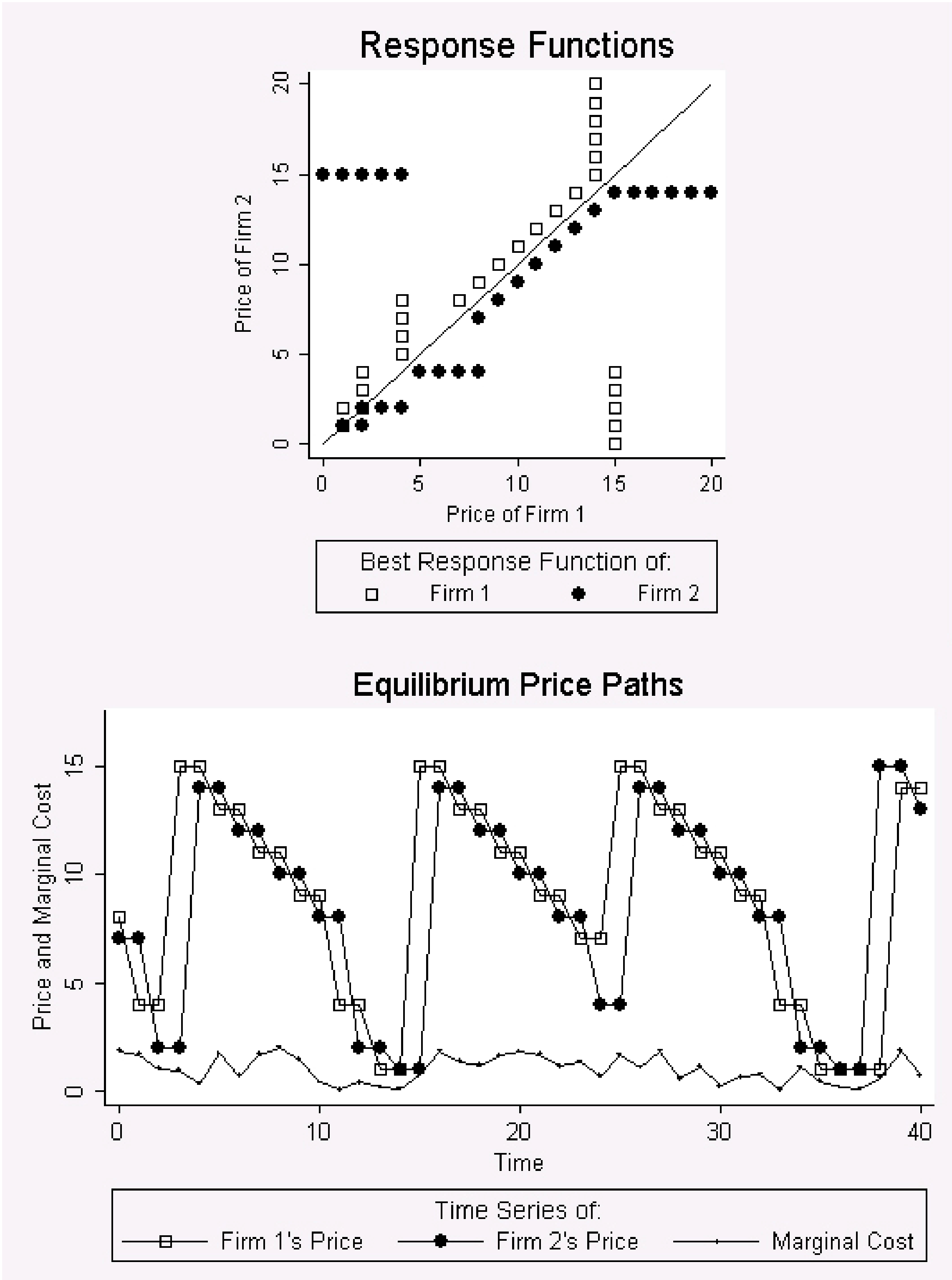

Figure 3. Symmetric Duopoly Model

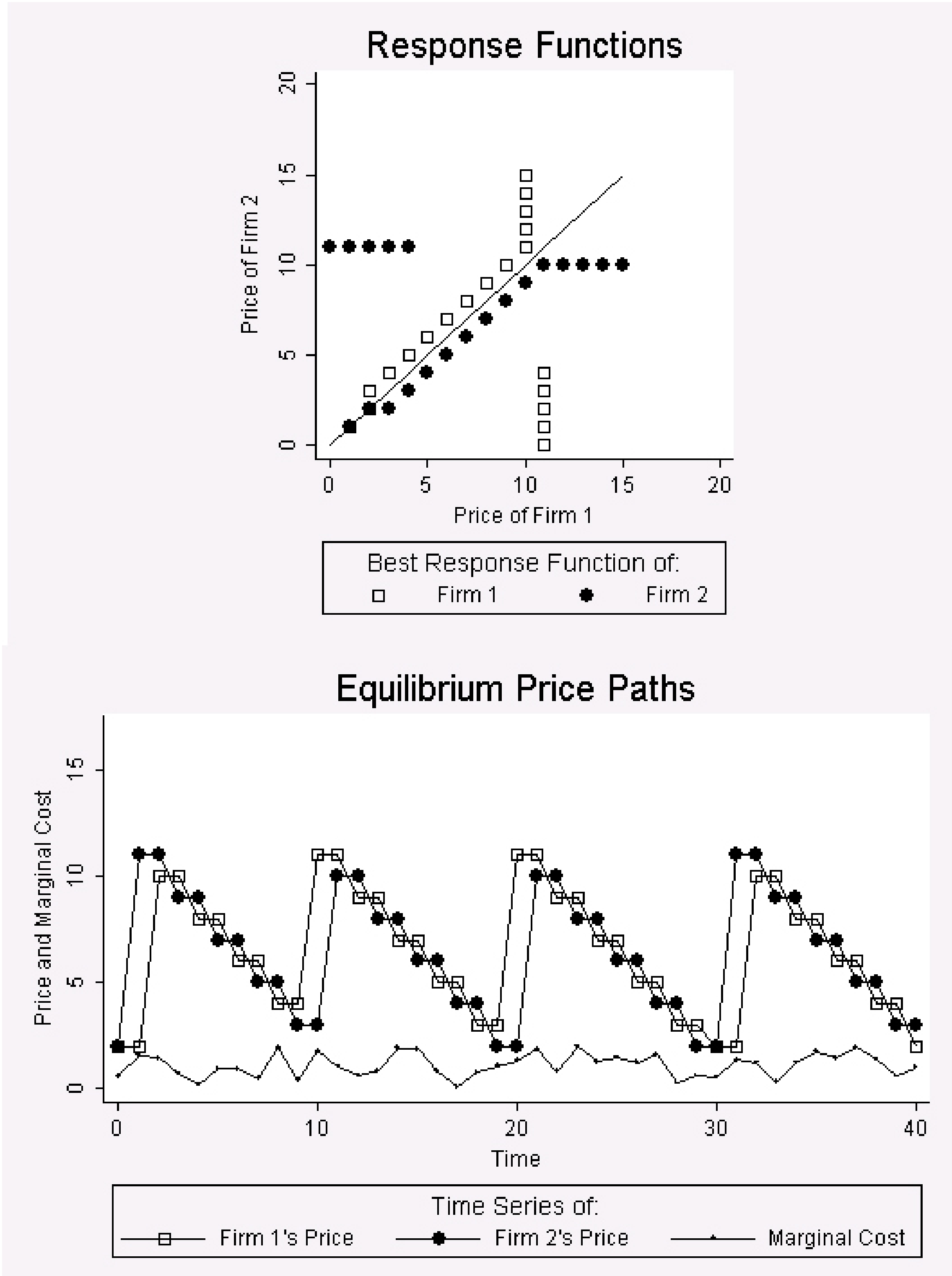

Figure 4. Symmetric Duopoly Model, Elastic Demand

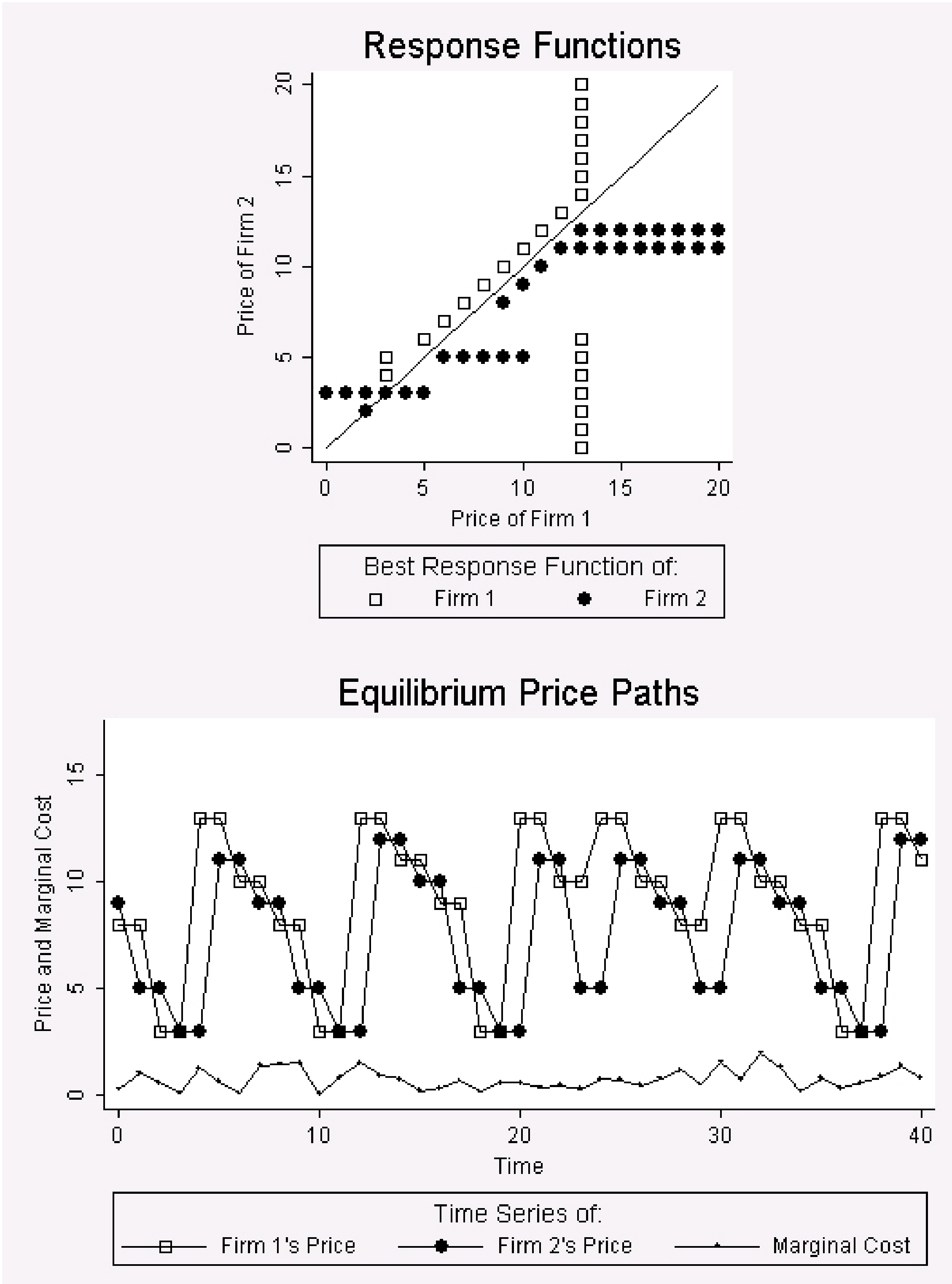

Figure 5. Asymmetric Duopoly Model

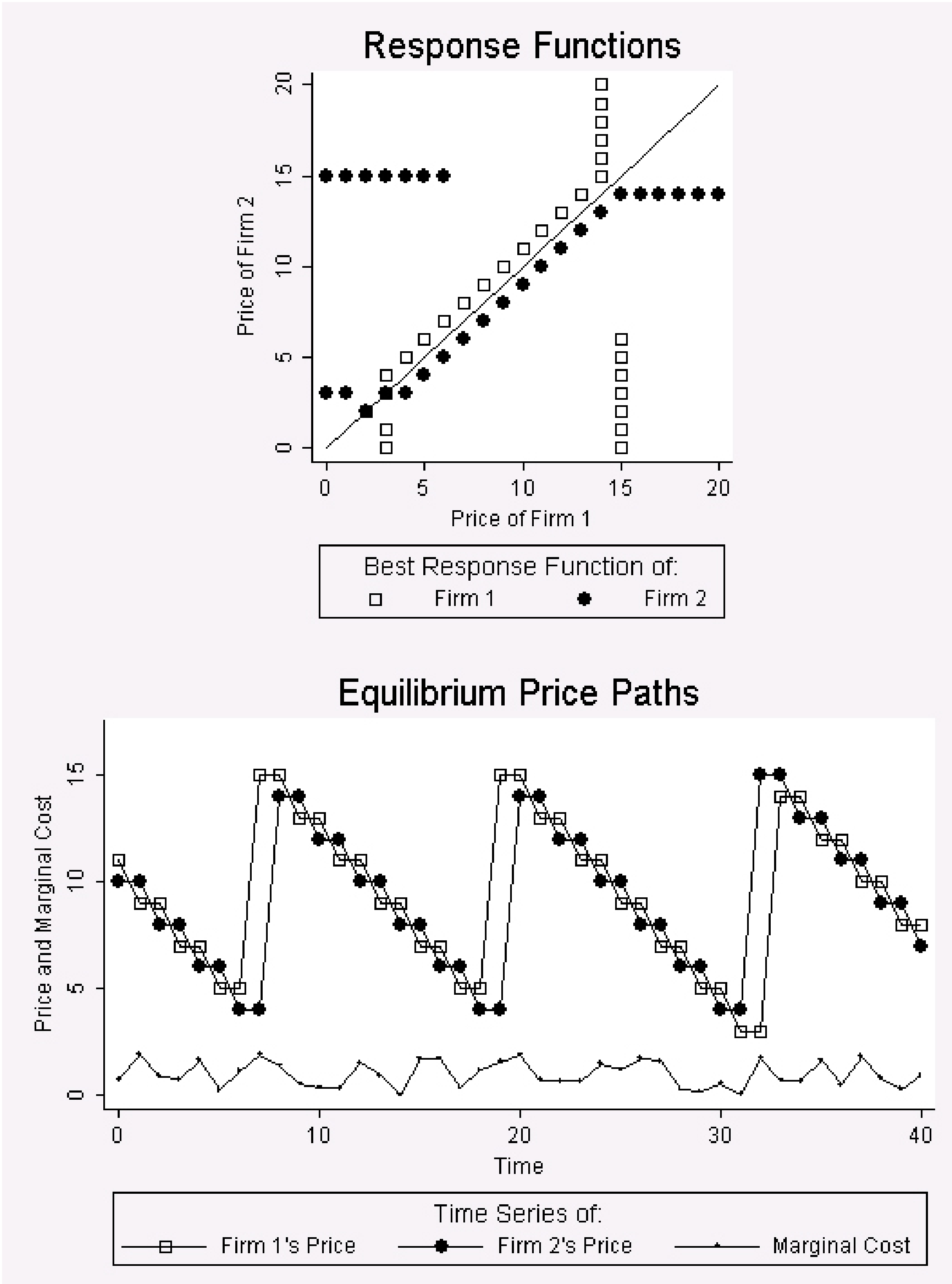

Figure 6. Symmetric Capacity Constraint Model, $K^i = 10$

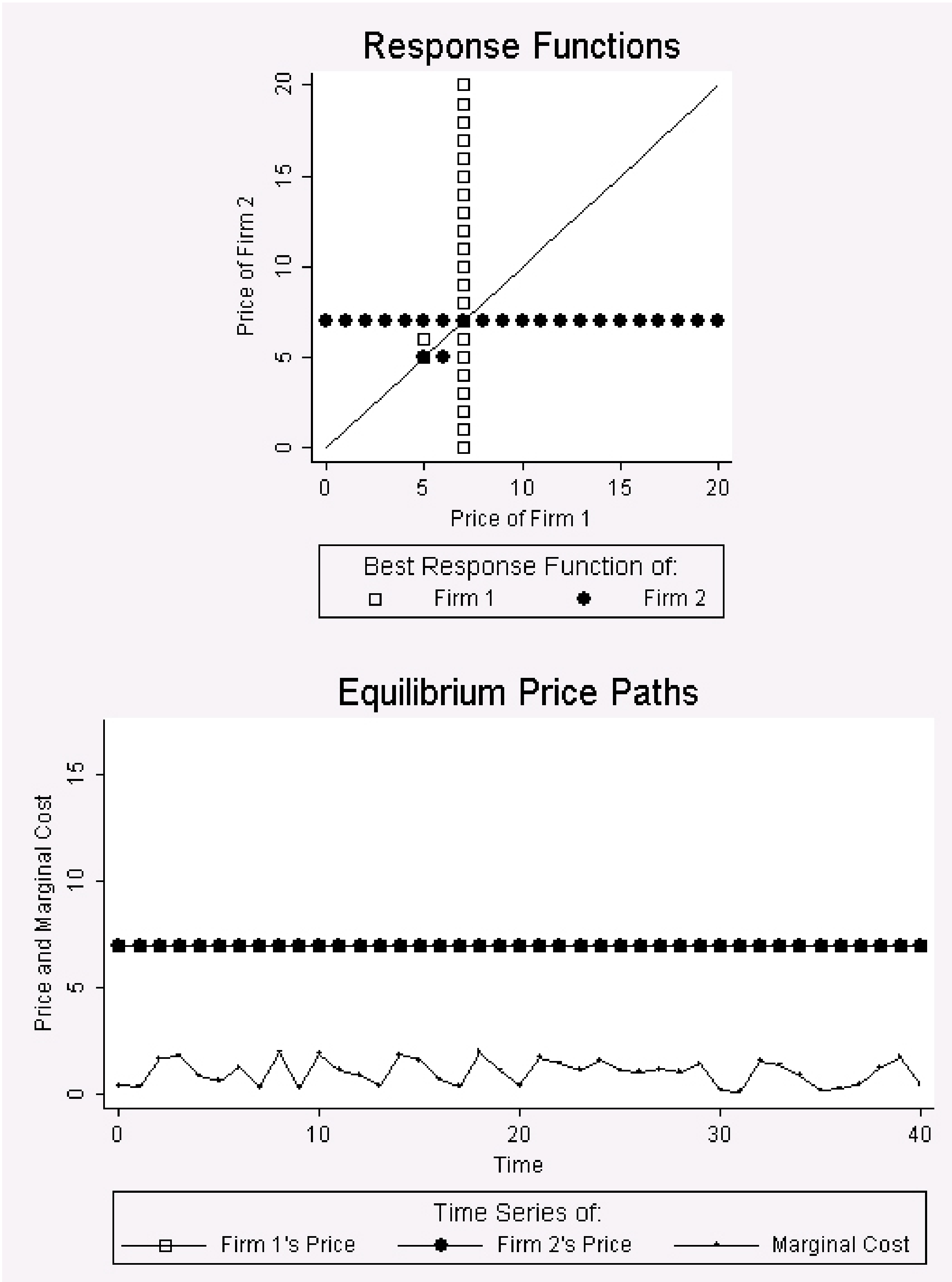

Figure 7. Symmetric Capacity Constraint Model, $K^i = 9$

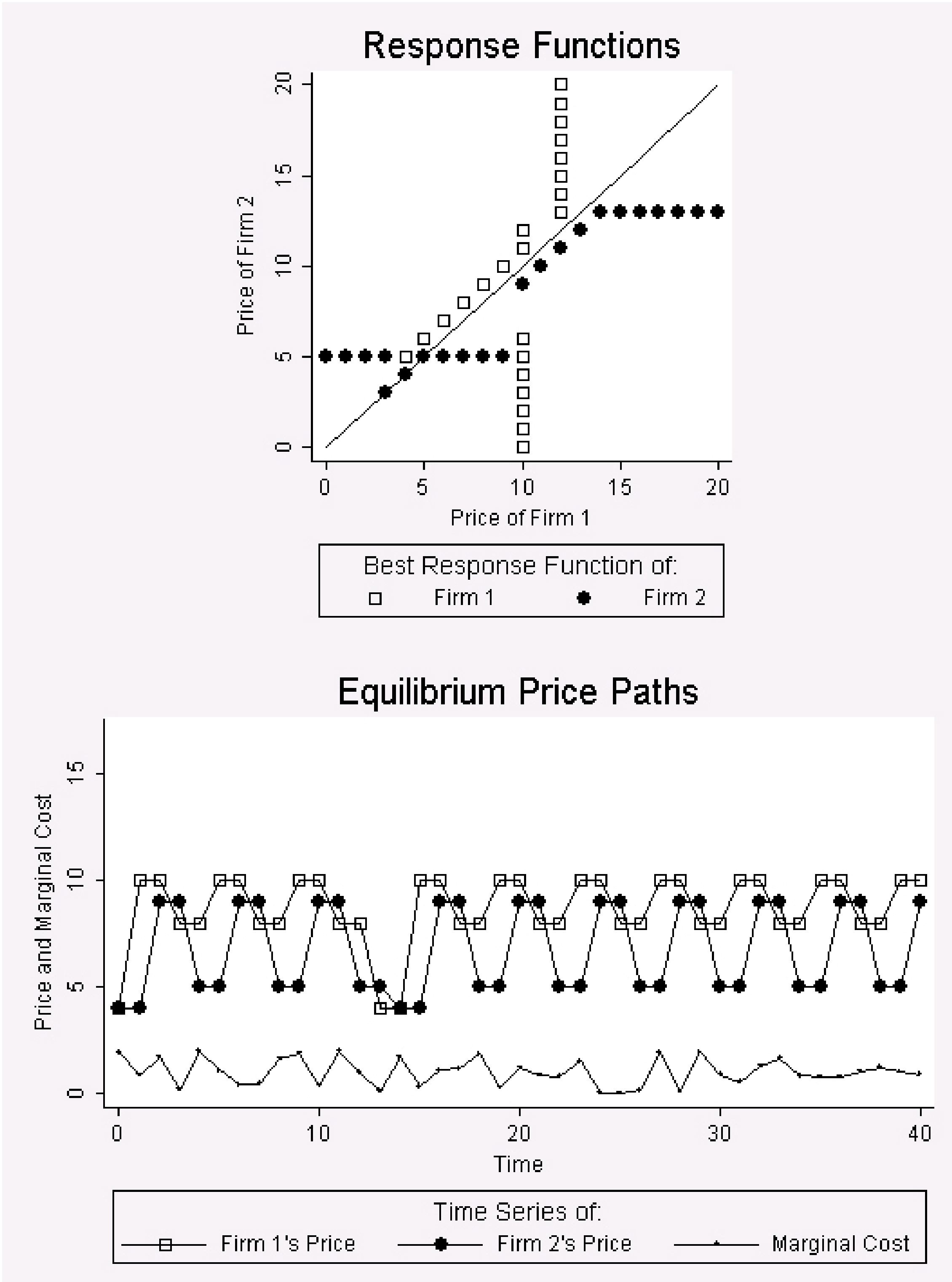

Figure 8. Asymmetric Capacity Constraint Model, $K^2 = 7$

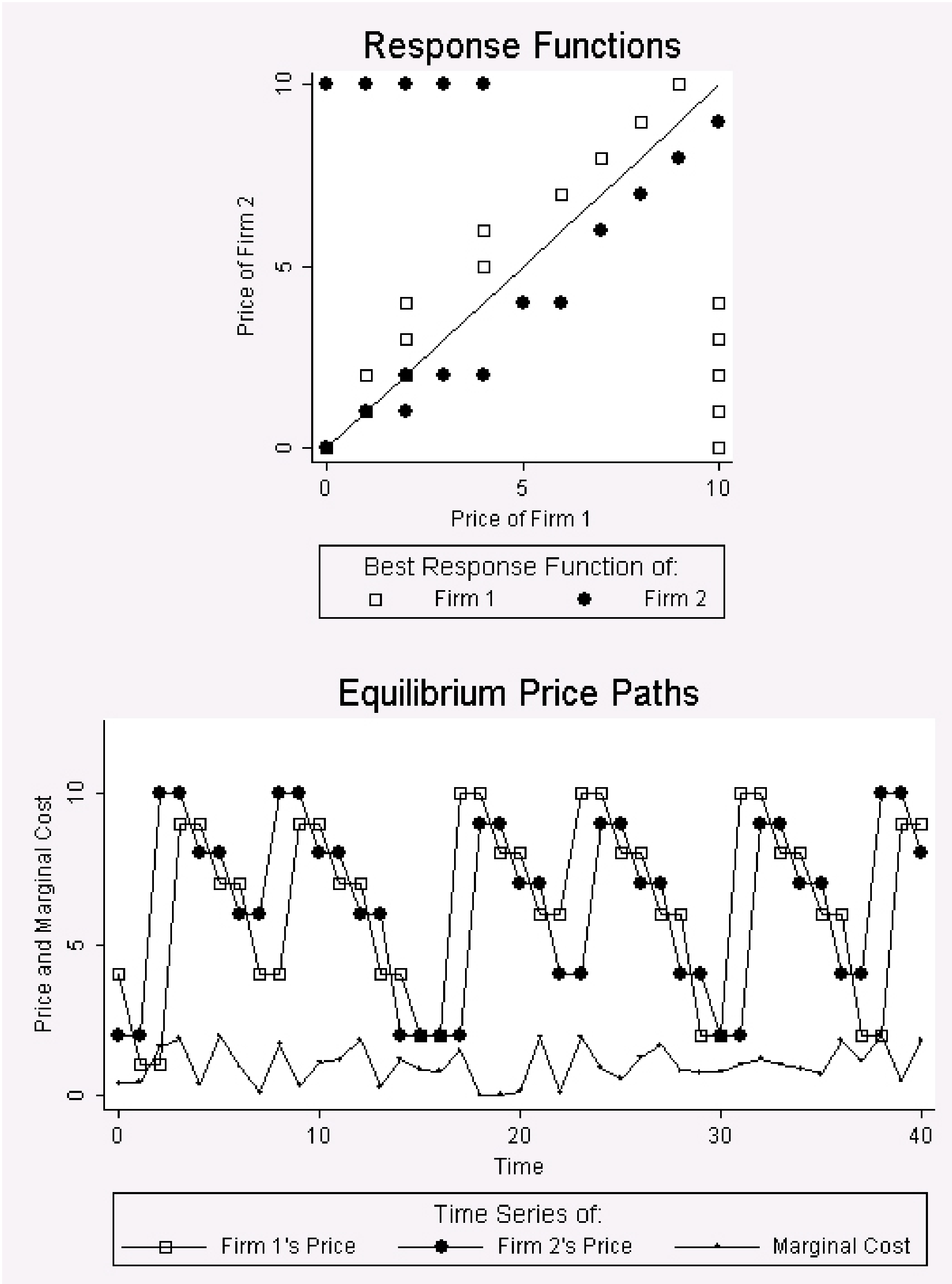

Figure 9. Differentiated Products Model, t = 1.1

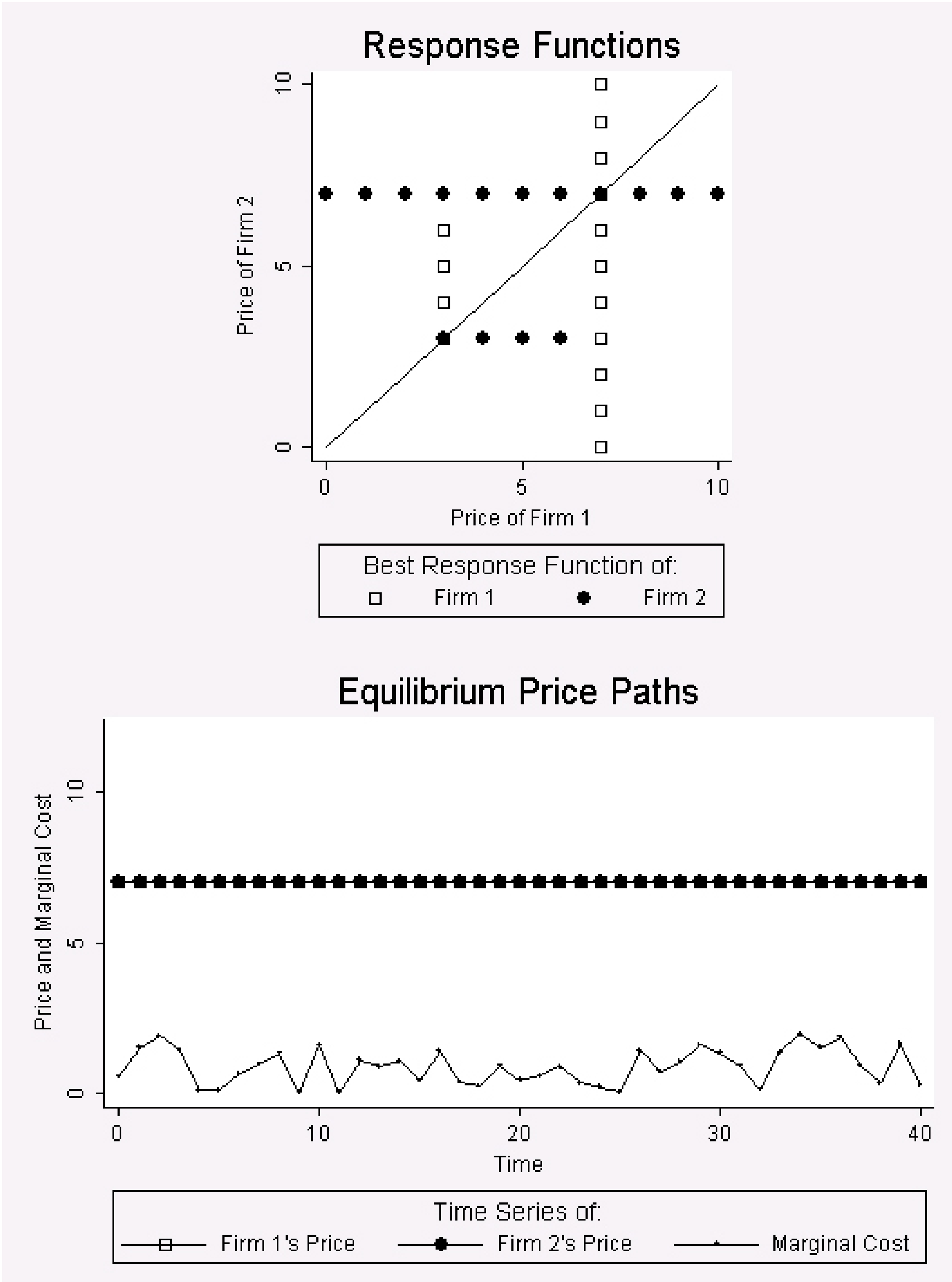

Figure 10. Differentiated Products Model, t = 1.25

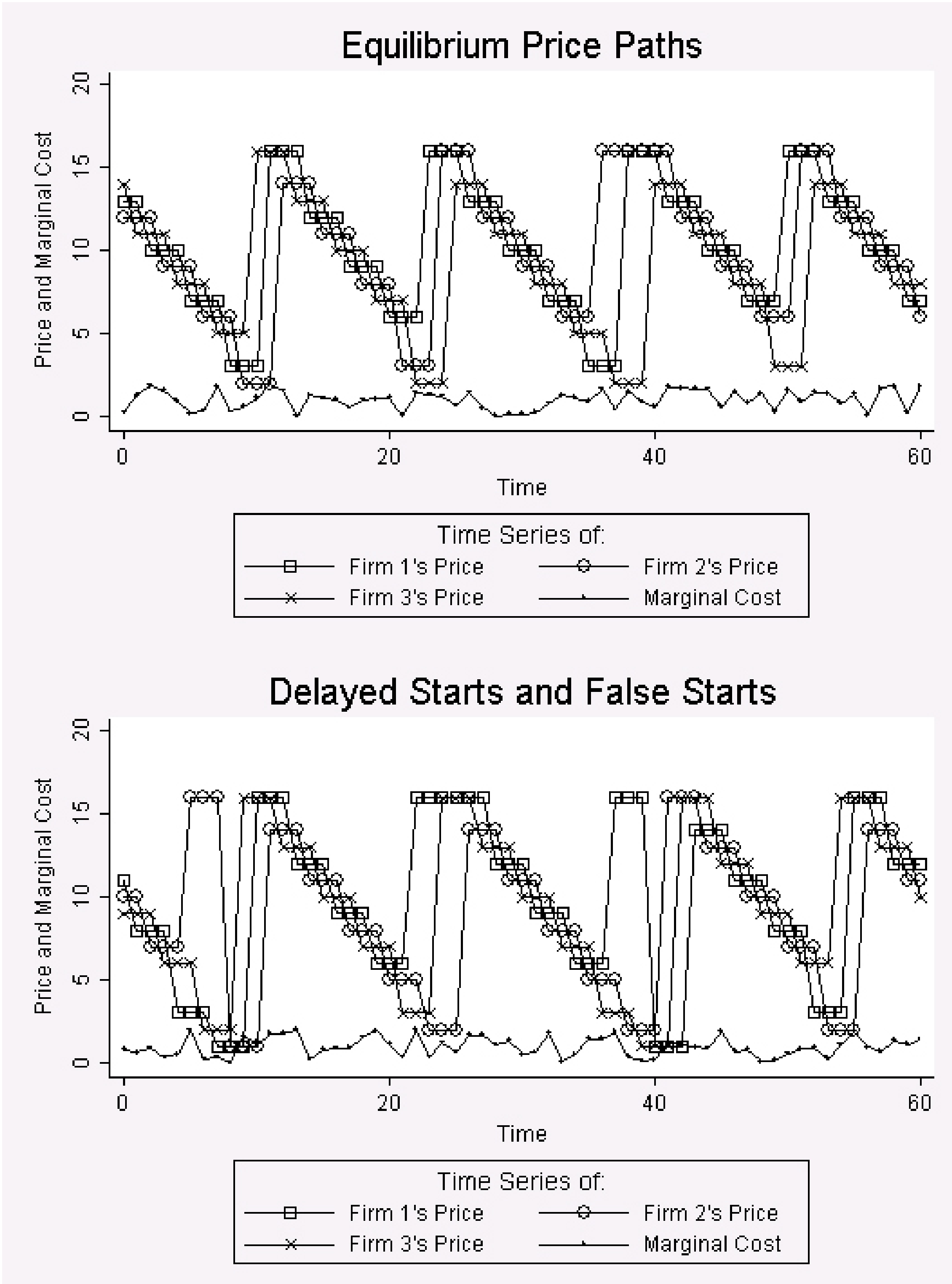

Figure 11. Triopoly Model